ISE International School of Economics and Social Sciences

KAZAKH BRITISH TECHNICAL UNIVERSITY

Review of the dissertation
"Asymptotic theory of regressions with asymptotically collinear regressors"
authored by Darkenbayeva Gulsim Spandiyarovna, submitted in fulfillment of the requirements for the degree of Doctor of Philosophy (PhD) for specialty 6D060100 – Mathematics

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The main subject of the dissertation is to study asymptotic properties of the linear model

(1) $y_t = \alpha + \beta L(t) + u_t$, $t = 1,...,n$,

where $L(t)$ is a slowly varying function and the error term is an unstable process

(2) $u_t = \rho u_{t-1} + v_t$

with $\rho = 1$ under the null hypothesis and v_t is a non-causal linear process.

The study of regressions (1) with slowly varying regressors and stable errors was initiated by P.C.B. Phillips in 2006. Mynbaev in his book of 2011 published at Wiley & Sons, USA, used his L_p-approximability theory to give a more complete treatment of (1). In the context of central limit theorems, the benefit of the L_p-approximability theory is that it allows one to study the variance of the limit distribution. Phillips in several papers also addressed the issue of hypothesis testing for linear regressions with unstable errors. However, he did not consider the combination (1)+(2). Uematsu (2014) attempted to study this combination but his proofs were incomplete. This is why I suggested this topic to Gulsim Darkenbayeva.

The topic turned out to be very difficult. It made necessary development of several theoretical tools: central limit theorems for linear and quadratic forms and L_p-approximability of some deterministic sequences arising in the context of regression (1). Convergence of some quadratic forms used in regression analysis did not fit the L_p-approximability theory and other tools had to be used for this purpose, such as the theorem by Cheng and Ho (2005) and convergence theorems from the book by Tanaka (1996).

Theorem 2.1 extends Mynbaev's theorem (2001) on convergence of quadratic forms to the case of asymmetric kernels. The most important results have been published in our joint paper in the Journal of Mathematical Analysis and Its Applications (2019), which is one of the leading forums for achievements in the theory of functions.

Overall, the dissertation became very advanced. Its results can be applied in many areas, not only in regression analysis. Gulsim Darkenbayeva did an excellent job in this research and her dissertation certainly satisfies the requirements to this kind of papers.

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